Preliminary Communication

About a possible occurrence of the Proudman resonance in the Adriatic

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In this paper the causes of the exceptional sea-level changes in the Adriatic, especially in the Vela Luka Bay, on June 21, 1978, are analysed. It is pointed out that one of the possible explanations is connected with the occurrence of the Proudman resonance. However, for a definitive hypothesis formulation, an adequate hydrodynamical numerical model will be needed, since the existing analytic solutions can not be satisfactorily applied to the basins that exist in the Adriatic.

INTRODUCTION

On the morning of June 21, 1978, at about 5 : 15 a. m., exceptional sea-level oscillations appeared in Vela Luka, a small town on the island of Korčula. Since at that time there was no tide gauge operating in the town, information about the phenomenon had to be obtained only on the basis of estimates from observers, taking into account the marks existing in Vela Luka today. Thus, the peak amplitudes of sea-level changes are evaluated at about 3 meters, while the period equals some 15 minutes; the oscillations lasted for three hours. The phenomenon caused great damage. It was established that at the same time similar sea-level changes occurred in a few other places in the Adriatic and that there were similar oscillations before and after those on the date mentioned.

Vela Luka is situated at the closed end of the Vela Luka Bay, located on the western side of the island of Korčula and opened to the west (Fig. 1). The Bay has a funnel-shaped form: its width at the closed end is approximately ten times smaller than the width at the opened end, and a similar ratio is valid for the depths.

Taking into consideration this position and configuration of the Bay, the question of the origin of the described sea-level oscillations can be approached.

According to the opinions put forward up to now, the cause should be sought in *a free progressive wave* that reached the Vela Luka Bay from the open sea and affected unusual sea-level changes through resonance — since its frequency was close to the natural frequency of the Bay. The descent of a free progressive wave on the open sea has been variously explained: while Zore-Armanda[1] pleads for tectonic changes, Hodžić[2] believes that atmospheric disturbances are of more importance.

In this paper the possibility of an essentially different explanation, connected with *a forced progressive wave* in the sea, will be studied. From the meteorological analyses the presence of a gravity wave in the atmosphere can be detected. This wave gives origin to a forced progressive wave in the sea, through the acting of atmospheric pressure over the sea. A forced wave is of small amplitude, until it meets a basin whose position and configuration are suitable for an occurrence of the Proudman resonance, *i. e.*, until the velocity

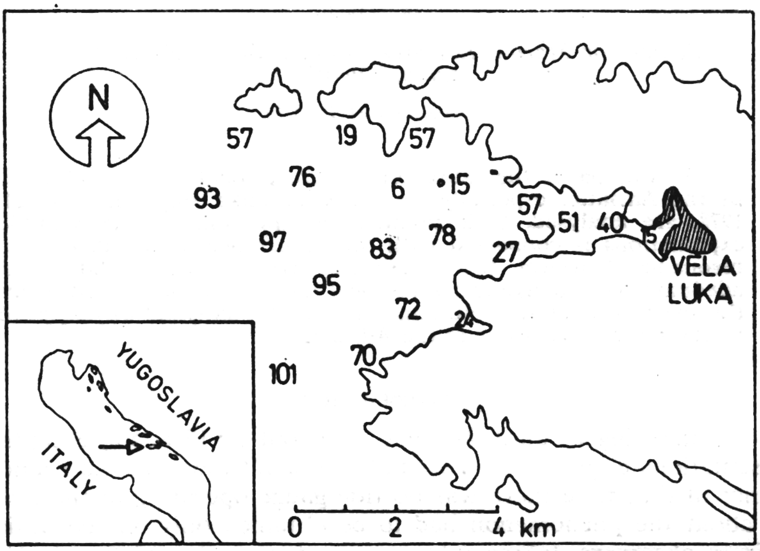

Fig. 1. The Vela Luka Bay — position and configuration; depths are given in meters.

of atmospheric disturbance (and the forced progressive wave) becomes equal to the velocity of a long wave in the sea. Then the amplitude of the forced wave increases, owing to the appearance of the Proudman resonance. Such resonance differs sharply from the one mentioned in the previous passage. While the Proudman resonance originates from the equalizing of the velocities of atmospheric disturbance and a long wave in the basin, the aforesaid resonance is the consequence of the frequency of the free progressive wave in the sea being equal to the natural frequency of the bay the wave is coming into.

In the remainder of this paper the movement of the atmospheric wave will be analysed, the basic explanation of the Proudman resonance will be given and, finally, the situation over the Adriatic on June 21, 1978 and the characteristics of the Vela Luka Bay will be examined in order to see if they fulfill the requirements for such a resonance.

ATMOSPHERIC WAVE

On the basis of analyses of meteorological situations concurrent with exceptional sea-level changes in the Adriatic, Hodžić[3–5] came to the conclusion that some characteristics are common for all such situations:

— southwestern flow in the upper layers of the atmosphere above the Adriatic;

— passage of cyclonic disturbance over the Adriatic;

— high-frequency oscillations of atmospheric pressure (on Hvar and Split stations).

Here the high-frequency oscillations of atmospheric pressure are of special interest. Therefore, such oscillations will be examined below, for June 21, 1978.

On that date the high-frequency oscillations appeared on the barograph records at five stations: Palagruža, Lastovo, Hvar, Dubrovnik—Ćilipi, Kardeljevo (Ploče). Figure 2 shows the positions of these meteorological stations

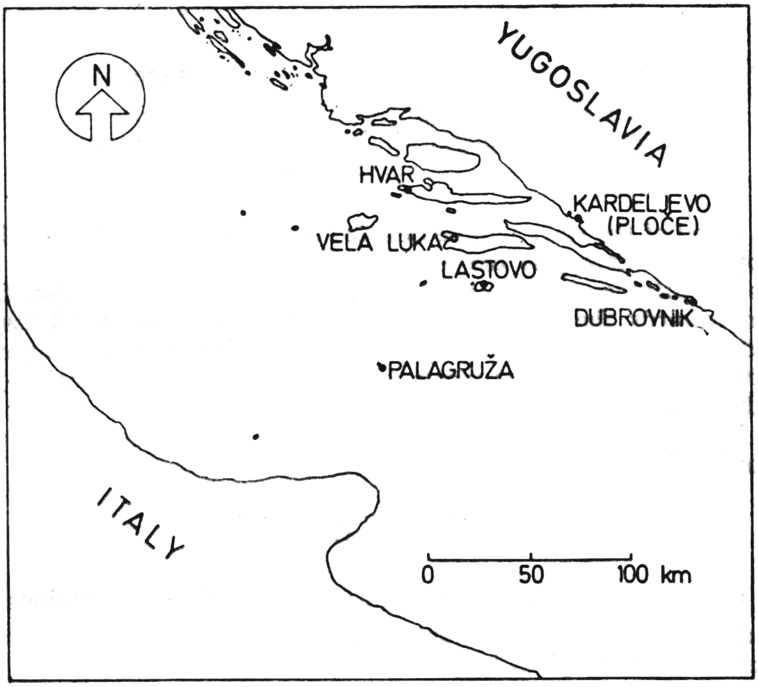

Fig. 2. Positions of meteorological stations on which the high-frequency oscillations of atmospheric pressure were registered on June 21, 1978. Also shown is the location of the Vela Luka Bay.

as they surround the Vela Luka Bay, while Table I gives the starting times of high-frequency oscillations on all five stations. Since the instruments operating on these stations were barographs, and not microbarographs of high sensitivity, starting times are most reliable parameters that can be obtained. The period of oscillations can only roughly be estimated at 10—60 minutes, while amplitude amounts to about 0.5 mm Hg. The oscillations lasted several hours.

The first question that should be answered is the question of the origin of these atmospheric pressure oscillations. From the differences in their starting times on various stations it can be concluded that they are the consequence of the passage of an gravity wave through the atmosphere above the Adriatic. On the assumption that this gravity wave was a plane wave of permanent form and that it was travelling at a constant speed along a straight line over the Adriatic, a simple method can be developed with the aim of obtaining the speed and direction of that wave — from the arrival-times data on various stations.

TABLE I

Starting times of the atmospheric pressure high-frequency oscillations, registered at five meteorological stations on June 21, 1978

Station	Time
Palagruža	4 : 30
Lastovo	5 : 15
Hvar	5 : 30
Dubrovnik — Ćilipi	5 : 50
Kardeljevo (Ploče)	6 : 00

The speed of the wave will be denoted by C, while direction will be determined by γ — where γ is the angle between the ray of the wave and the parallels of latitude. Angle γ will be measured counterclockwise, from the direction towards the east. Moreover, let δ_i be the distance between two meteorological stations, γ_i the angle between the line δ_i and the parallels of the latitude (angle γ_i will be measured in the same way as angle γ) and let Δt_i be the computed difference between arrival times on stations connected with δ_i. From fundamental geometrical analysis it can easily be shown that the following is valid:

$$\Delta t_i = \frac{\delta_i \cdot \cos(\gamma_i - \gamma)}{C}$$

Finally, the measured difference between arrival times on various stations will be designated by $\overline{\Delta t_i}$. The numerical values of all the known parameters mentioned earlier are given in Table II.

TABLE II

Values of the parameters δ_i, γ_i, $\overline{\Delta t_i}$

Parameter / Pair of stations	δ_i [m]	γ_i	$\overline{\Delta t_i}$ [s]
Palagruža — Lastovo (i = 1)	67000	38°	2700
Lastovo — Hvar (i = 2)	58000	130°	900
Hvar — Dubrovnik/Ć (i = 3)	160000	—24°	1200
Dubrovnik/Ć — Kardeljevo (Ploče) (i = 4)	83000	143°	600

If now the following condition is stated:

$$f(C, \gamma) = \sum_{i=1}^{4} (\Delta t_i - \overline{\Delta t_i})^2 = \sum_{i=1}^{4} \left(\frac{\delta_i \cdot \cos(\gamma_i - \gamma)}{C} - \overline{\Delta t_i} \right)^2 = \min$$

the values of the unknown parameters C, γ can be obtained. Graphic analysis, on the basis of Fig. 3, gives:

$$C = 22 \text{ m/s}$$

$$\gamma = 58^0$$

If these amounts are used as approximate values in the procedure of equalizing the indirect measurements of the same accuracy, as described by Čubranić[6], the values already obtained for the unknown parameters will be confirmed, the mean errors of the unknown parameters will be obtained and, lastly, the check of the computation will be carried out. This procedure gives:

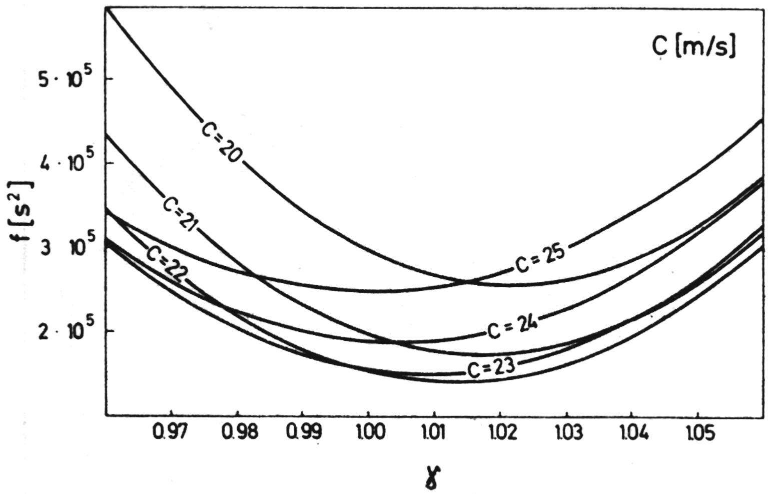

Fig. 3. Graphic presentation of the function f (C, γ).

$$C = 22 \pm 2 \text{ m/s}$$

$$\gamma = 58 \pm 2^0$$

The dispersion of the results is to be expected, first because of the errors connected with the measurement of the parameter $\overline{\Delta t_i}$, and second because of the assumptions introduced for the atmospheric wave. It is interesting that the dispersion is not larger. This fact can be taken as a confirmation of the assumptions mentioned.

A clearer insight into the maning of the above results can be obtained by means of Fig. 4, where arrival times for the atmospheric wave are presented, as they follow from the measurements and theoretical analysis. It can be seen that the wave travelled approximately from the southwest to the northeast above the Adriatic, going over the numerous bays and channels of the eastern Adriatic and also over the Vela Luka Bay. It is of special interest

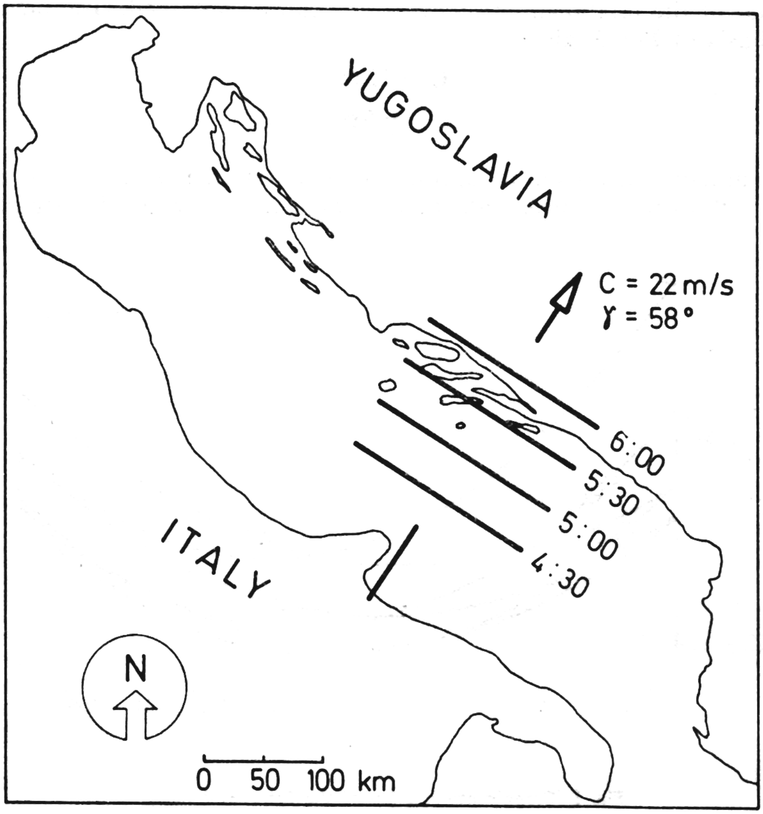

Fig. 4. Arrival times of the atmospheric wave that crossed the Adriatic on June 21, 1978.

that the passing of the atmospheric wave over Vela Luka coincided with the appearance of exceptional sea-level oscillations; the interaction of atmosphere and sea will be studied on the basis of theoretical expressions presented below.

THE PROUDMAN RESONANCE

A long straight channel of constant width will be examined here; the depth h of the water is supposed to be uniform. The coordinate system is placed in such a way that the x axis corresponds to the axis of the channel and the whole problem is reduced to one spatial dimension.

Introducing the usual approximations for long waves, and representing the acting of atmospheric pressure by means of equilibrium elevation $\bar{\zeta}$, the equations of motion and continuity take the form:

$$\frac{\partial u}{\partial t} + g \frac{\partial}{\partial x} (\zeta - \bar{\zeta}) = 0$$

$$h \frac{\partial u}{\partial x} + \frac{\partial \zeta}{\partial t} = 0 \tag{1}$$

where ζ is the elevation, u is the mean value of the current taken along the vertical profile, while g represents the acceleration of gravity.

It will be assumed that the atmospheric pressure disturbance travels over the channel at a constant speed U along the x axis, consequently that it is proportional to the function F (x — Ut), so that the equilibrium elevation is given by:

$$\bar{\zeta} = F(x - Ut) \tag{2}$$

In that case, the solution of Eq. 1 is, according to Proudman[7,8]:

$$\zeta = \frac{1}{1 - U^2/c^2} F(x - Ut)$$

$$u = \frac{U}{h(1 - U^2/c^2)} F(x - Ut) \tag{3}$$

where $c = \sqrt{gh}$ is the velocity of long waves in the basin. From Eq. 3 it can be seen that resonance — the Proudman resonance — will occur when the velocity U of the atmospheric disturbance becomes equal to the velocity c. Under this condition the above expressions for ζ and u tend to infinity, and the solution fails. As pointed out by Lamb[9], the interpretation of this is that the amplitude of the motion becomes so great that the fundamental approximations are no longer justified.

These results will now be applied to the Vela Luka Bay. If the orientation of the Bay and the characteristics of the atmospheric disturbance movement are taken into account (these latter according to the results in the previous paragraph), it can easily be shown that the following is valid:

$$U = \frac{C}{\cos(\gamma - 10^0)} = 29 - 37 \text{ m/s}$$

Therefore, for the occurrence of the Proudman resonance depths from the 86—140 m interval are needed. And depths from this interval can be found in front of the Vela Luka Bay (Fig. 1). With that the occurrence of the Proudman resonance would be confirmed, if the numerous approximations leading to Eq. 3 were not taken into account. Since the Vela Luka Bay does not satisfy some of these simplifications, they should be analysed before a final conclusion is made. Three basic facts that should be kept in mind are:

— the existence of the closed end of the basin;
— the real configuration of the basin;
— the influence of friction.

Here the first of these elements will be examined.

Let it be supposed that the channel is closed at $x = 0$, but extends infinitely in the positive direction along the x axis. The boundary condition reads $u = 0$ at $x = 0$. Let equilibrium elevation be of the form:

$$\bar{\zeta} = f\left(t - \frac{x}{U}\right) \tag{4}$$

Then, in accordance with Eq. 4, atmospheric disturbance travels again at a constant speed U along the x axis. Proudman[7, 8] gives the solution of Eq. 1, if the problem is put in this way:

$$\zeta = \frac{1}{1 - U^2/c^2}\left[f\left(t - \frac{x}{U}\right) - \frac{U}{c} f\left(t - \frac{x}{c}\right)\right]$$
$$u = \frac{U}{h\,(1 - U^2/c^2)}\left[f\left(t - \frac{x}{U}\right) - f\left(t - \frac{x}{c}\right)\right] \tag{5}$$

At $x = 0$:

$$\zeta = \frac{f(t)}{1 + U/c} = \frac{\bar{\zeta}}{1 + U/c}$$

Hence, the Proudman resonance can occur when the velocity U becomes equal to the velocity c, but only if $U < 0$, *i. e.*, when the atmospheric disturbance travels towards the closed end of the basin. For the area characterized by dominant movements of atmospheric disturbances in the west-east direction, this condition selects — as favourable for the occurrence of the Proudman resonance — only those bays that are opened to the west.

If once more the attention is turned to the Vela Luka Bay (Fig. 1), it can easily be seen that the Proudman resonance can occur in this Bay, under condition:

$$C = \sqrt{gh}\cos(\gamma - 10^\circ)$$

With $h = 54$ m, which is the mean depth of the Vela Luka Bay according to Sterneck[10], and with $\gamma = 56 - 60^\circ$, the theoretical values of the speeds C fall in an interval of 15 — 16 m/s. Correspondence with the values measured (20—24 m/s) is not specially good, but a disagreement could be expected for the simple analytic solution which takes into account the existence of the closed end only, and not the real configuration of the basin.

In future work the real configuration of the Vela Luka Bay should be incorporated into the analysis. This can be achieved with the aid of an adequate hydrodynamical numerical model, which at present is under preparation at the »Ruđer Bošković« Institute. Finally, friction should be included into the model, and that will probably lead to the conclusion that the funnel-shaped form of the Bay is of some importance for the occurrence of exceptional sea-level changes.

CONCLUSIONS

Preliminary analysis put forward in this paper showed that the possibility of the occurrence of the Proudman resonance exists in the Adriatic, especially in the Vela Luka Bay. Also, it was shown that only a hydrodynamical numerical model will enable the hypothesis to be formulated, the hypothesis that would connect the appearance of exceptional sea-level changes in Vela Luka with the forced progressive wave and the Proudman resonance. That hypothesis will make the organization of measurements possible, leading — along with further theoretical investigations — to the definitive explanation of the Vela Luka phenomenon. Naturally, during measurements and theoretical research also the conception about the influence of the free progressive wave should be taken into account, according to the opinions of other authors.

If the occurrence of the Proudman resonance will be confirmed, it would not be the first such phenomenon registered in the Adriatic; already Caloi[11] found a similar phenomenon in the Trieste Bay, but the amplitudes measured there were much smaller — probably owing to the different configuration of the basin.

At the end, it should be pointed out that up to now only the first phase of the Vela Luka phenomenon has been considered. After the initial impulses most probably standing waves appear in the Bay, and they also deserve a thorough study. As an introduction, it should be mentioned that the only tide-gauge measurements performed to date in Vela Luka were analysed by Sterneck[10], and that he found the following periods for standing waves:

— 37.8 minutes (the whole basin),

— 12.6 minutes (the inner part of the basin).

It seems that these results have completely been forgotten till now, as well as the fact that the tide gauge was operating for a while in Vela Luka, in 1906.

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IZVOD

O mogućoj pojavi Proudmanove rezonancije u Jadranu

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U radu se analiziraju uzroci pojave izuzetnih oscilacija vodostaja u Jadranu, specijalno u zaljevu Vela Luka 21. lipnja 1978. godine. Jedno od mogućih tumačenja povezano je s pojavom Proudmanove rezonancije. Međutim, za definitivno postavljanje hipoteze bit će potrebno razraditi odgovarajući hidrodinamički numerički model, budući da postojeća analitička rješenja ne odgovaraju bazenima kakvi se javljaju u Jadranu.